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NOVEL STRUCTURE TO THE COUPLED NONLINEAR MACCARI'S SYSTEM BY USING MODIFIED TRIAL EQUATION METHOD

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Abstract. In this article, we obtain some new travelling wave solution of the coupled nonlinear Maccari's system. The purpose of this article is to research new exact travelling wave solutions of the coupled nonlinear Maccari's system by applying to the Modified Trial Equation Method (MTEM). This method is very efficient and suitable for solving nonlinear differential equations and system of equations. The solutions that we find have not in the literature until recently.

Keywords: *modified trial equation method, coupled nonlinear Maccari's system, travelling wave solutions.*

AMS Subject Classification: *00A69.*

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1. Introduction

The differential equations are widely used in modeling physical phenomenons, fluid dynamics, biology, chemistry and so on. It's not easy to obtain the analytical solutions of nonlinear differential equations that describe mathematical models. In the literature, there are many effective analytical and numerical methods improved by researchers such as the Kudryashov Method, the (G'/G) -expansion method, the modified simple equation method, the modified sine-cosine method, homotopy perturbation method, homotopy analysis method, extended trial equation method and so on [4-8,11,13-15, 18].

The purpose of this article is to put into practice MTEM to construct the new exact travelling wave solutions for the coupled nonlinear Maccari's system. Maccari's system is a kind of nonlinear evolution equations that are often presented to describe the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optic, and so on. Some types of Maccari system have been investigated lots of researchers [1-3, 9, 12]. Zhang used Exp-function method for seeking exact solutions of Maccari's system [19]. The first integral method is applied to the coupled nonlinear Maccari's system to obtain exact solution [17]. The novel (G'/G)-expansion method has been studied for finding the exact solutions of the (2+1)-dimensional coupled integrable nonlinear Maccari system [10]. In [16], the new extension of the (G'/G)-expansion method for solving the coupled nonlinear

Maccari's system is given and thus hyperbolic function solutions, trigonometric function solutions and rational function solutions have been obtained.

2. Fundamental facts of the modified trial equation method

The modified trial equation method (MTEM) consists of four steps. In this sub-section we present these steps.

Step 1. We consider partial differential equation in two variables and a dependent variable u:

$$P(u,u_t,u_x,u_{xx},u_{xxx},...)=0, \tag{1}$$

and take the wave transformation,

$$u(x,t)=u(\xi), \xi = kx-ct, \tag{2}$$

where k and c are constants can be determined later. By substituting equation (2) into equation (1), a nonlinear partial differential equation (NODE) is converted as following:

$$N(U,U',U'',U''',...)=0. \tag{3}$$

Step 2. Take trial equation as follows:

$$U'=\frac{F(u)}{G(u)}=\frac{\sum_{i=0}^{n}a_iu^i}{\sum_{j=0}^{l}b_ju^j}=\frac{a_0+a_1u+a_2u^2+\cdots+a_nu^n}{b_0+b_1u+b_2u^2+\cdots+b_lu^l}, \tag{4}$$

$$U''=\frac{F(u)\left(F'(u)G(u)-F(u)G'(u)\right)}{G^3(u)}, \tag{5}$$

where $F(u)$ and $G(u)$ are polynomials. Substituting above relations into (3) yields an equation of polynomial $\Omega(u)$ of u:

$$\Omega(u)=\rho_su^s+\cdots+\rho_1u+\rho_0=0. \tag{6}$$

According to the balance principle, we can get a relationship between n and l. We can compute some values of n and l.

Step 3. Let the coefficients of $\Omega(u)$ all be zero will yield a system of the algebraic equations

$$\rho_i=0, i=0,1,2,\cdots,s. \tag{7}$$

By solving this system, we will thus determine the values of $a_0,a_1,\ldots,a_n$ and $b_0,b_1,\ldots,b_l$.

Step 4. Reduce (4) to the elementary integral form,

$$\pm(\mu-\mu_0)=\int\frac{G(u)}{F(u)}du. \tag{8}$$

Using a complete discrimination system for polynomial of $F(u)$, we solve Equation (8) with the help of Mathematica 9 and classify the exact solutions to Equation (3). For better explication of results obtained in this way, we can plot two and three dimensional surfaces of the solutions obtained by using suitable parameters.

3. Application

We consider the coupled nonlinear Maccari's system [16] described as follows:

$$\begin{cases} iQ_t + Q_{xx} + RQ = 0, \\ iS_t + S_{xx} + RS = 0, \\ iN_t + N_{xx} + RN = 0, \\ R_t + R_y + (|Q+S+N|^2)_x = 0. \end{cases} \tag{9}$$

In order to research exact solution of system (9), we apply following transformations,

$$\begin{cases} Q(x,y,t) = u(x,y,t)e^{ik(kx+\alpha y+\lambda t+l)}, \\ S(x,y,t) = v(x,y,t)e^{ik(kx+\alpha y+\lambda t+l)}, \\ N(x,y,t) = w(x,y,t)e^{ik(kx+\alpha y+\lambda t+l)}, \end{cases} \tag{10}$$

where k, α, l and λ are constants to be determined later, l is an arbitrary constant. Substituting Eqs. (10) into system (9) and using the transformation

$$u = U(\xi), v = V(\xi), w = W(\xi), R = R(\xi), \xi = x + \beta y - 2kt, \tag{11}$$

where β is a constant, we perform required arrangement, system (9) becomes as follow:

$$U'' - (\lambda + k^2)U - \frac{(1+r_1+r_2)^2}{\beta - 2k}U^3 = 0. \tag{12}$$

According to balance principle highest order nonlinear terms of U'' and U^3 in (12), we get the following relationship

$$2n - 2l - 1 = 3 \Rightarrow n = l + 2. \tag{13}$$

This resolution procedure is performed and we can obtain some analytical solutions as follows.

Case1. If we take $l = 1$, $n = 3$ we have

$$U' = \frac{F(u)}{G(u)} = \frac{\sum_{i=0}^{n} a_i u^i}{\sum_{j=0}^{l} b_j u^j} = \frac{a_0 + a_1 u + a_2 u^2 + a_3 u^3}{b_0 + b_1 u}, \tag{14}$$

$$\begin{aligned} U'' &= \frac{F(u)\left(F'(u)\,G(u) - F(u)\,G'(u)\right)}{G^3(u)} \\ &= \frac{\left(a_1 + 2a_2 u + 3a_3 u^2\right)\left[\left(a_1 + 2a_2 u + 3a_3 u^2\right)\left(b_0 + b_1 u\right) - \left(a_1 + 2a_2 u + 3a_3 u^2\right)\left(b_1\right)\right]}{\left(b_0 + b_1 u\right)^3}, \end{aligned} \tag{15}$$

where $a_3 \neq 0$ and $b_1 \neq 0$. When we use U' and U'' in (12), we get a system of algebraic equations for (12). Solving this system (12) by using Wolfram Mathematica 9 yields the following coefficients

$$\begin{aligned} a_3 &= \frac{-ib_1\left(1+r_1+r_2\right)}{\sqrt{4k-2\beta}}, \\ \lambda &= \frac{k^2 b_1\sqrt{2k-\beta}+i\sqrt{2}a_1\left(1+r_1+r_2\right)}{\sqrt{2k-\beta}\,b_1}, a_0=0, a_2=0, b_0=0. \end{aligned} \tag{16}$$

By considering Eq.(15) for Eq.(8), we have

$$\pm\left(\mu-\mu_0\right)=\int\frac{b_1u+b_0}{a_3u^3+a_2u^2+a_1u+a_0}\,du. \tag{17}$$

Integrating Eq.(17) with Eq.(16) by using Wolfram Mathematica 9, and then, by doing some simplifications such as $\eta_0=0$, we obtain the solution in the form of complex trigonometric function to the Eq.(9) as following:

$$u(x,y,t)=\sqrt{\frac{a_1\sqrt{4k-2\beta}}{-ib_1\left(1+r_1+r_2\right)}}Tan\left[\frac{\xi}{b_1}\sqrt{\frac{-ia_1b_1\left(1+r_1+r_2\right)}{\sqrt{4k-2\beta}}}\right]. \tag{18}$$

Substituting Eq. (18) into system (10) we have the functions of $Q(x,y,t), S(x,y,t), N(x,y,t)$ and also $R(x,y,t)$.

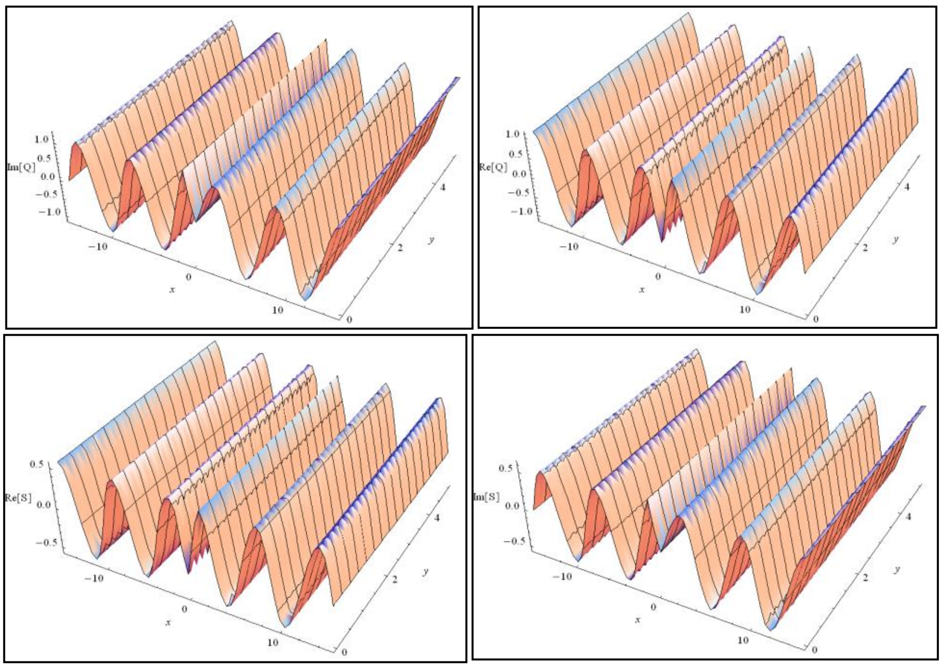

Figure1. The 3D graphics that have been hold by Eq.(18) for
$\alpha=0.02,\ \lambda=0.03,\ l=0.04,\ k=1,\ \beta=0.01,\ t=0.01,$
$r_1=0.5,\ r_2=0.9,\ -15<x<15,\ 0<y<5.$

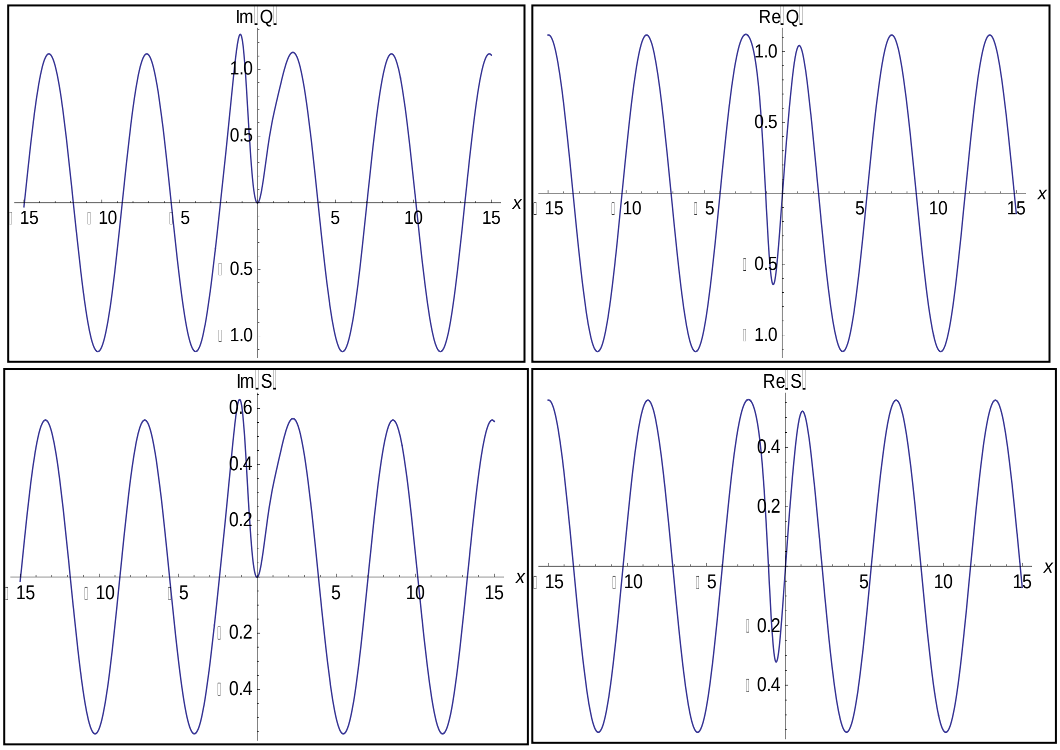

Figure2. The 2D graphics that has been hold by Eq.(18) for
$\alpha = 0.02,\ \lambda = 0.03,\ \ l = 0.04,\ \ k = 1,\ \beta = 0.01,\ t = y = 0.01,$
$r_1 = 0.5,\ r_2 = 0.9,\ -15 < x < 15,\ \ 0 < y < 5.$

3. Conclusion

In this article, the modified trial equation method has been successfully applied to the coupled nonlinear Maccari's system. We have found new travelling wave solutions to Eq.(9). We have given some figures to describe the behavior of the obtained solutions of Eq. (9). According to these results, this method is reliable and effective for these models.

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